Conference Proceedings

Educating the educators: international approaches to scaling-up professional development in mathematics and science education

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A Virtual Mathematics Laboratory in support of educating educators in inquiry-based style -
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The paper deals with a Virtual School Mathematics Laboratory (VirMathLab) being developed at the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences (IMI-BAS). Various ways of using it in mathematics education are presented, with emphasis on the implementation of the Inquiry Based Learning (IBL) by means of dynamic geometry software. Strategies for integrating it with the education of educators in mathematics and information technologies are discussed. The paper deals with the specifics of seminars for teachers organized in the frames of recent European projects (InnoMathEd, Fibonacci, DynaMat, Mascil, Scientix2, KeyCoMath) the authors have been involved in for implementing the IBL in mathematics education.

1 Preparing the ground for mathematics explorations on a large scale

The idea of preparing and developing a repository of e-resources which could be used in support of inquiry based mathematics education at different levels and forms led to the establishment of a Virtual School Mathematics Laboratory (VirMathLab) at IMI-BAS (http://www.math.bas.bg/omi/cabinet/), Chehlarova et. al. (2014).

Important feature of the VirMathLab is that it will be dynamically enriched by resources developed within educational projects with two-way links. For instance, the dynamic Snowflakes file(http://www.math.bas.bg/omi/mascil/task-Snowflakes-bg.html) is linked to the scenario “Let us make a snowflake” within the MaSciL project which in turn refers to the Snowflakes file (Figure 1).

![Figure 1: The sites of the VirMathLab and Mascil at IMI-BAS](image)

Every dynamic file could be considered as a half-baked e-resource to be used as a means for: providing conditions for explorations, visualization of the solutions, testing and self-testing, creating and formulating mathematics
problems, preparation of didactic resources on paper, solving practical problems with a specific precision, motivation for mathematical or programming activities, acquiring skills for working with a specific software, forming of competencies for working with a text (mathematical and CS alike), development of algorithmic thinking, etc. (Figure. 2).

Figure 2: Examples of dynamic scenarios in VirMathLab

The dynamic files are available in three formats, the choice depending on the goals and the technical skills of the users:
http://www.math.bas.bg/omi/cabinet/content/bg/ggb/d22054.ggb
http://www.math.bas.bg/omi/cabinet/content/bg/html/d22054.html
They could be used in various ways depending on the level of preparation, as described below in the context of a specific example.

2 A 3-day seminar form

Such a seminar form proved to be very efficient, the third day being 1-2 months after the first 2, in which the teachers present their own projects (individually or as a team). During the presentation the participants discuss the project, suggest modifications and improvements. The questions posed by the teachers are targeted to their specific audience of students and the answers come from the rest of the teachers and us, the authors, in the role of teacher educators. Another important feature of these seminars is that they are binary, i.e. led by two people simultaneously. The dialogue type of communication among the seminar leaders is both a model to follow and usually provokes similar style of communication among the teachers. The presence of two sources of information and professional experience enables (i) different aspects of consideration of the object being studied, (ii) comparison between different points of view, and (iii) a specific analysis of a given situation. Various models of decision making and team work are demonstrated.

Our experience shows that one can work effectively in homogeneous and heterogeneous groups alike in terms of technical skills, experience in working with specific learning environments, age range, etc. What is crucial though is that there are at least two teachers per school (in mathematics and in
informatics/IT). The synergy between their competencies is a guarantee for a noticeable progress in implementing the innovation in school.

The seminars provide teachers with knowledge about a sensible use of educational software and other resources (dynamic scenarios and manipulatives) together with innovative educational strategies (e.g. inquiry based learning). It is essential that the teachers enter the skin of researchers so that they could implement with self-confidence the IBL in their practice. This includes leaving them to explore and find out on their own the potential of a software environment (new to them) in a limited time, to solve a specific mathematics problem in inquiry style, to solve specific didactic cases.

The crucial part of the courses is for the participants to experience different stages and levels of IBL. The teachers work on pedagogical problems related with: reformulating of math problems in IBL style so as to enhance the development of specific key competences; formulating their own math problems reflecting real-life situations, not solvable with the current math knowledge of the students but allowing for explorations by means of dynamic geometry models leading to an acceptable approximation of the solution; studying and proposing methods for tackling problems which are unstructured, or whose solutions are insufficient or redundant; solving “traditional problems” with “non-traditional” data, for which the use of a computing device is necessary; applying game-design thinking so as to engage better the students in the problem solving; formulating more relevant evaluation criteria for the students’ achievements; assessment of learning resources in terms of formation and development of IBL skills and key competences; project-based work with presentation of the results.

We specify three levels of competencies for working with dynamic constructions: (i) using them as offered by somebody else, (ii) modifying them for a specific purpose, and (iii) creating them from scratch.

For example, the file at http://www.math.bas.bg/omi/cabinet/content/bg/html/d14007.html or http://www.math.bas.bg/omi/cabinet/content/bg/ggb/d14007.ggb could be used in the given format for forming the notion of a fraction, when learning the basic property of the fractions, etc. (Figure 3).

![Figure 3: A dynamic model for forming the notion of a fraction](image)

This file could support the good understanding of the nature of fractions, as well as the self-checking of corresponding knowledge and skills. The activities could take the form of a game. In a relatively short time every student can first
solve a specific problem and then create a new problem for the next participant. It is sufficient that the student moves the point M so that the green rectangle corresponds to the fraction in the condition (3/7 in the case of Figure 4).

![Figure 4: Locating M so that the colored part is 3/7 of the whole rectangle](image)

If needed the students could ask the program for help (checking the box помош) which leads to the division of the rectangle in the corresponding number of equal parts (7 in our case, Figure 5).

![Figure 5: Asking for help (checking the box помош)](image)

Including an answer box (отговор in Bulgarian) enables the feedback (Figure 6).

![Figure 6: Asking for the answer (checking the box отговор)](image)

To pose another problem the students should first hide the help box and the answer box (by unchecking them) and then choose new values for the numerator and denominator by means of the sliders (Figure 7).

![Figure 7: Posing a new problem](image)

Thus, when working on specific examples, the teachers get familiar with elements of the software and develop their digital competence. When carrying out an experiment with teachers and students, we were surprised by the results of the teachers, especially of those working in the primary school. More
than 70% would commit essential errors when solving similar problems on a sheet of paper. It turned out that after 10 min work with the file discussed above their results were significantly better. Their abilities to form knowledge and skills for estimating the result matter even more today when the calculations are often performed by means of computing devices. This format of the files is helpful also for self-checking. In some cases, the VirMathLab contains a series of files with modified conditions, e.g. intervals of the admissible values of the numerator and denominator, using a circle instead of a rectangle, etc.

In most of the cases however it is appropriate for the teachers to modify the file according to specific goals. For instance, they can change the end points of the sliders in the above case from [1, 10] to [1, 20] (as shown in Figure 8).

![Figure 8: Modification of a dynamic file - changing the range of the parameters](image)

Another option is to insert mathematical text in the file (several fractions in this case, such that their depiction by means of the sliders could lead to formulating a hypothesis, Figure 9).

![Figure 9. Modification of a dynamic file – introduction of mathematical text](image)

The file could be modified so as to be used for solving the reverse problem – given the denominator and a colored part of a rectangle to figure out the numerator of the fraction corresponding to the colored part (Figure 10).
The main use of the dynamic files is the possibility for explorations and formulation of hypotheses. When working files with dynamic constructions which could be investigated, it is common to perform additional calculations, to observe results, relations, measures. In such case it is convenient to show them on the screen. To achieve a better precision the appearance of the numbers might need to be tuned accordingly (e.g. more digits after the decimal point to be shown). In other cases additional constructions are helpful. Sometimes it is a good idea for the teachers to change only the design of the objects, e.g. if the constructions is to be displayed on a big screen, the lines would appear better if thicker, in a darker colour, etc.

According to our experience, to make use of the full potential of a dynamic resource developed in support of IBL the teachers should possess competencies at second level at least (modifying the files for a specific purpose).

The seminars under consideration have made use of VirMathLab (http://www.math.bas.bg/omi/cabinet/) and resources developed in the frames of the projects Fibonacci (http://www.math.bas.bg/omi/Fibonacci/), DynaMat (http://www.dynamathmat.eu/), mascil (http://www.math.bas.bg/omi/mascil/).

3 Other form of teacher education

Other PD forms in our practice include:

- PD events (seminars and workshops) in the frames of conferences

The key feature of these events is that the teachers have an active role and act as partners in a research team – they share their good practices in oral or poster presentations (sometimes jointly with their students), work in groups on specific tasks and present their ideas to the rest of the participants. Typical examples include the Scientix National Conference within the National seminar Inquiry Based Mathematics Education (http://www.math.bas.bg/omi/nso/), the Dynamic Mathematics in Education conference (http://www.math.bas.bg/omi/dmo/), the seminars within the Spring conferences of UBM, the regional conferences organized by UBM sections, the International UNESCO workshop QED Chehlarova, 2012, Sendova, 2015.

Joint research sessions on a specific problem – e.g. face-to-face work on the Problem of the Month within the mascil project (Figure 3), possibly followed by a virtual meeting with teachers and students from the partner countries.

- Building and developing competences necessary for the students to participate in new types of mathematics contests, e.g. Mathematics with a
computer, Theme of the month (Figure 11) (Kenderov and Chehlarova, 2014), (Chehlarova and Kenderov, 2015), Branzov, (2015), Gachev, (2015).

Figure 11: “Theme of the month”: a long-term activity on a math problem modeling a real-life situation

- **Mathematics performances** – events raising the awareness of the general public about the role of mathematics for enhancing children’s scientific curiosity and endeavour to learn (Chehlarova and Sendova, 2013). The examples include: Performance at the History Museum in Stara Zagora, organized by the UBM section in the town, performances during the Researchers’ Nights (2011-2014), Science festivals (in Italy, Romania, Greece) (Figure 12). It is important to note that the teachers act as multipliers of the IBL ideas during these events as well – they participate with their students, and occasionally lead the performance.

Figure 12. Posters for math performances within Science Fairs
• **Individual work with teachers** – it includes support for the development of lessons, educational materials, mathematical fests, course projects, peer reviews, and preparation of a pedagogical experiment.

4 **Conclusions**

Although it is early to claim that the inquiry based learning of mathematics is widely used in Bulgaria, at least we could claim that our team has contributed to creating a community of teachers who implement and spread further this style of learning.

These teachers participate in pedagogical experiments not only as a *reality-proof of researchers* but as members of a research team, implement, modify and develop from scratch educational resources in support of IBL, share their good practices at seminars and conferences, and in professional journals. Some of them organize public events at a school and regional level for popularizing the Inquiry based mathematics education, for demonstrating the connection between mathematics and the world of work.

The students’ motivation, their changed attitude towards learning of mathematics, and the recognition of their achievements are just one aspect of the teachers’ success. The progress students make, their raising self-confidence, the feeling that they belong to a community of learners are the crucial factor which supports and enriches the community of teachers introducing, implementing and disseminating the inquiry based mathematics education (IBME) by means of specialized dynamic software.

Here follow what two teachers (the so-called *mascil multipliers*) have expressed after getting a special recognition of their activities in implementing and disseminating IBL:

**Elisaveta Stefanova**: *The best is not the award itself but the fact that we, the teachers feel members of a community of soul mates...*

**Neli Stoyanova**: *When I decide to give problems appropriate for IBL, I don’t think of the curriculum and the syllabus, I leave my students to inquire, to think, to combine, to create and to surpass me!... The award brings a great satisfaction since many colleagues expressed their wish for future collaboration, and their interest is not less important. The decision of the jury is recognition for my long-term activities, a confirmation that the IBL ideas are well-received in Bulgaria and abroad, and most importantly – that the resources developed by my students do matter.*

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References


Chehlarova, T., E. Sendova, 2013, The Mathematics performance – a social game or educational technology, 42nd Spring Conference of the UBM, Sofia, p. 159-166. (in Bulgarian)


Kenderov, P., T. Chehlarova, 2014, The contest “Viva Mathematics with a computer” and its role for the development of digital competence of the students (in Bulgarian), Shumen, MATTEX. pp. 3-10


Sendova, E., 2015, The MaScil seminar and the poster session in the frames of QED’14 a forum for innovative educational ideas (in Bulgarian), in Kovatcheva, E., Sendova, E (eds.) UNESCO International Workshop Quality

Bulgarian site of Mascil: http://www.math.bas.bg/omi/mascil/index.html


National seminar in education: The Inquiry Based Mathematics Education

Virtual School Mathematics Laboratory (VirMathLab)